High Q Resonant Cavities for Microwave Testing

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Formulas and charts are given which aid the design of right circular cylinder cavity resonators operating in the $TE\ 01n$ mode, which yields the highest Q for a given volume. The application of these to the design of an echo box radar test set is shown, and practical considerations arising in the construction of a tunable cavity are discussed.

Introduction

A TUNABLE high Q resonant cavity is a particularly useful tool for determining the over-all performance of a radar quickly and easily[1].* Further, since it uses the radar transmitter as its only source of power, it can be made quite portable. When a high Q cavity is provided with two couplings, one for the radar pickup and the other to an attenuating device, crystal rectifier and meter which serve for tuning the cavity not only can an indication of over-all performance be obtained but other useful information as well. For example, the transmitter frequency can be measured; calibration of the crystal affords a rough measure of the transmitter power; and an analysis of the spectrum can be made by plotting frequency versus crystal current. This information is of particular importance in radar maintenance.

The Q required for this purpose is quite high, comparable to that obtained from quartz crystals in the video range. For this reason, such cavities have many additional possibilities for use in microwave testing equipment and microwave systems. For example, they may form component parts of a narrow band filter, or be used as discriminators for an oscillator frequency control.

Resonant cavities are of two general types—tuned and untuned. A tuned cavity is designed to resonate in a single mode adjustable over the radar frequency range. An untuned cavity is of a size sufficient to support a very large number of modes within the working range. Both are useful, but the tuned variety can give more information about the radar and hence has been more widely used.

While a tuned cavity may be a cylinder, parallelepiped or sphere (or even other shapes), the first of these has been most thoroughly explored by us. It offers the possibility of utilizing the anomalous circular mode, described by Southworth[2] in his work on wave guides, which permits the attainment of high Q's in quite a small size. In addition, it is easier to construct a variable length cylinder than a variable sized sphere.

* Superscripts refer to bibliography.

Due to their interesting properties the history of resonators of the cavity type in which a dielectric space is enclosed hy a conducting material, goes back many years. In 1893 J. J. Thompson[3] derived expressions for resonant frequencies of the transverse electric modes in a cylinder. Lord Rayleigh[4] published a paper in 1897 dealing with such resonant modes. The early work was almost entirely theoretical hut some experiments were carried out in 1902 hy Becker[5] at 5 and 10 centimeters. In recent years, the suhject has been fairly thoroughly investigated (at least theoretically) for several simple shapes.

However, many of the presentations are highly mathematical with considerahle space devoted to proofs; the results which would he most useful to an engineer are thus sometimes ohscured. The purpose of this paper is to present certain engineering results together with information upon the application of the tunahle cylindrical cavity to radar testing.

Definitions and Fundamental Formulas

Modes

By fundamental and general considerations, every cavity resonator, regardless of its shape, has a series of resonant frequencies, infinite in numher and more closely spaced as the frequency increases. The total number N of these having a resonant frequency less than f is given approximately hy:[6]

$$N = \frac{8\pi}{3c^3} V f^3 \tag{1}$$

in which

V = volume of cavity in cuhic meters.

c = velocity of electromagnetic waves in the dielectric in meters per second.

f = frequency in cycles per second.

With each resonance there is associated a particular standing wave pattern of the electromagnetic fields, which is identified hy the term "mode."

In right cylinders (ends perpendicular to axis) the modes fall naturally into two groups, the transverse electric (TE) and the transverse magnetic (TM). In the TE modes, the electric lines everywhere lie in planes perpendicular to the cylinder axis, and in the TM modes, the magnetic lines so lie. Further identification of a specific mode is accomplished by the use of indices.

The MS Factor

With the cylinder further restricted to a loss-free dielectric and a nonmagnetic surface, there is associated with each mode a value of Q (quality factor)[7] which depends on the conductivity of the metallic surface, on the

frequency and on the shape of the cylinder, e.g. whether it is circular or elliptical, and whether it is slender or stubby. The quantity $\frac{Q\delta}{\lambda}$, however, depends only upon the mode and shape of the cylinder and has been referred to as the mode-shape (MS) factor. In this formula, δ refers to skin depth as customarily defined[8], and λ is wavelength in the dielectric, as given by $\lambda = \frac{c}{f}$; both δ and λ are in meters.

Fundamental Formulas

Expressions for standing wave patterns and $Q\frac{\delta}{\lambda}$ are given in Table I, for right rectangular, circular and full coaxial cylinders*. The table is virtually self-explanatory, but a few remarks on mode designation are needed. The mode indices are l, m, n following the notation of Barrow and Mieher.[9] In the rectangular prism they denote the number of half-wavelengths along the coordinate axes. For the other two cases they have an analagous physical significance with l related to the angular coordinate, m to the radial and n to the axial.

In the elliptical cylinder, a further index is needed to distinguish between modes which differ only in their orientation with respect to the major and minor axes; these paired modes are termed even and odd, and have slightly different resonant frequencies.[10] In the circular cylinder they have the same frequency, a condition which is referred to as a degeneracy (in this case, double); that is, in the circular cylinder, odd and even modes are distinguishable only by a difference in their orientation within the cylinder with reference to the origin of the angular coordinate. In Table I, the field expressions are given for the even modes; those for the odd modes are obtained by changing $\cos l\theta$ to $\sin l\theta$ and $\sin l\theta$ to $\cos l\theta$ everywhere.

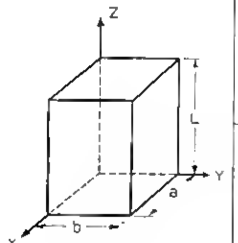
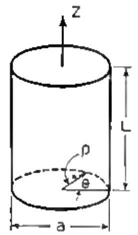
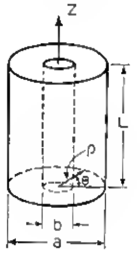
The value of N in the table is based on counting this degeneracy as a single mode; counting even and odd modes as distinct will nearly double the value of N, thus bringing it into agreement with the general equation (1). The distinction between even and odd modes is of limited importance in practical applications, and will not be further mentioned.

In Table I, the *mks* system of units is implied. The notation is in general accordance with that used in prior developments of the subject. For engineering applications, it is advantageous to reduce the results to units in ordinary use and to change the notation wherever this leads to a more obvious association of ideas. For these reasons, in what follows attention

* The elliptic cylinder (closely allied to the circular cylinder of which it is a generalization) is omitted as the necessary functions are not widely known or easily available.

This table belongs between pages 410 and 411 of the article HIGH Q RESONANT CAVITIES FOR MICROWAVE TESTING by I. G. Wilson, C. W. Schramm and J. P. Kinzer in the Bell System Technical Journal, July, 1946. Through an error, it was omitted from the printed journal.

TABLE I.—*Formulas for Cavity Resonators—Fields, Resonant Frequencies and Mode Shape Factors for Rectangular Prism, Circular Cylinder and Full Coaxial*

TYPE OF CAVITY & CO-ORDINATE SYSTEM	MODE	FIELD EQUATIONS *	DEFINITIONS	RESTRICTIONS ON ℓ, m, n
RECTANGULAR PRISM 	TM	$E_x = \sqrt{\frac{\mu}{\epsilon}}\frac{k_1k_3}{k^2}\cos k_1x \sin k_2y \sin k_3z$ $E_y = \sqrt{\frac{\mu}{\epsilon}}\frac{k_2k_3}{k^2}\sin k_1x \cos k_2y \sin k_3z$ $E_z = -\sqrt{\frac{\mu}{\epsilon}}\frac{k_1^2+k_2^2}{k^2}\sin k_1x \sin k_2y \cos k_3z$ $H_x = -\frac{k_2}{k}\sin k_1x \cos k_2y \cos k_3z$ $H_y = \frac{k_1}{k}\cos k_1x \sin k_2y \cos k_3z$ $H_z = 0$	$k_1 = \frac{\ell\pi}{a}$ $k_2 = \frac{m\pi}{b}$ $k_3 = \frac{n\pi}{L}$ $k^2 = k_1^2+k_2^2+k_3^2$ $\lambda = \frac{2\pi}{k}$ ℓ, m, n = INTEGRAL INDICES IDENTIFYING THE MODES. MAY ASSUME THE VALUE ZERO, SUBJECT TO RESTRICTIONS GIVEN IN ADJOINING COLUMN	$\ell > 0$ $m > 0$
	TE	$E_x = -\sqrt{\frac{\mu}{\epsilon}}\frac{k_2}{k}\cos k_1x \sin k_2y \sin k_3z$ $E_y = \sqrt{\frac{\mu}{\epsilon}}\frac{k_1}{k}\sin k_1x \cos k_2y \sin k_3z$ $E_z = 0$ $H_x = \frac{k_1k_3}{k^2}\sin k_1x \cos k_2y \cos k_3z$ $H_y = \frac{k_2k_3}{k^2}\cos k_1x \sin k_2y \cos k_3z$ $H_z = -\frac{k_1^2+k_2^2}{k^2}\cos k_1x \cos k_2y \sin k_3z$		$\ell + m > 0$ $n > 0$
CIRCULAR CYLINDER 	TM	$E_\rho = -\sqrt{\frac{\mu}{\epsilon}}\frac{k_3}{k}J_\ell'(k_1\rho)\cos \ell\theta \sin k_3z$ $E_\theta = \sqrt{\frac{\mu}{\epsilon}}\ell\frac{k_3}{k}\frac{J_\ell(k_1\rho)}{k_1\rho}\sin \ell\theta \sin k_3z$ $E_z = \sqrt{\frac{\mu}{\epsilon}}\frac{k_1}{k}J_\ell(k_1\rho)\cos \ell\theta \cos k_3z$ $H_\rho = -\ell\frac{J_\ell(k_1\rho)}{(k_1\rho)}\sin \ell\theta \cos k_3z$ $H_\theta = -J_\ell'(k_1\rho)\cos \ell\theta \cos k_3z$ $H_z = 0$	$k_1 = \frac{2r_{\ell m}}{a}$ $k_3 = \frac{n\pi}{L}$ $k^2 = k_1^2+k_3^2$ $\lambda = \frac{2\pi}{k}$ ℓ, m, n = DEFINED AS FOR RECTANGULAR PRISM $r_{\ell m}$ = m^{th} ZERO OF $J_\ell(x)$ FOR TM MODES $r_{\ell m}$ = m^{th} ZERO OF $J_\ell'(x)$ FOR TE MODES	$m > 0$
	TE	$E_\rho = -\sqrt{\frac{\mu}{\epsilon}}\ell\frac{J_\ell(k_1\rho)}{k_1\rho}\sin \ell\theta \sin k_3z$ $E_\theta = -\sqrt{\frac{\mu}{\epsilon}}J_\ell'(k_1\rho)\cos \ell\theta \sin k_3z$ $E_z = 0$ $H_\rho = \frac{k_3}{k}J_\ell'(k_1\rho)\cos \ell\theta \cos k_3z$ $H_\theta = -\ell\frac{k_3}{k}\frac{J_\ell(k_1\rho)}{k_1\rho}\sin \ell\theta \cos k_3z$ $H_z = \frac{k_1}{k}J_\ell(k_1\rho)\cos \ell\theta \sin k_3z$		$m > 0$ $n > 0$
FULL COAXIAL  $\frac{b}{a} = \eta$	TM	SAME AS FOR CIRCULAR CYLINDER, BUT SUBSTITUTE: $Z_\ell(k_1\rho)$ FOR $J_\ell(k_1\rho)$ $Z_\ell'(k_1\rho)$ FOR $J_\ell'(k_1\rho)$ WHERE $Z_\ell(k_1\rho) = J_\ell(k_1\rho) - AY_\ell(k_1\rho)$ $Z_\ell'(k_1\rho) = J_\ell'(k_1\rho) - AY_\ell'(k_1\rho)$	SAME AS CIRCULAR CYLINDER, EXCEPT: $r_{\ell m}$ = m^{th} ZERO OF $[J_\ell(\eta x)\,Y_\ell(x) - J_\ell(x)\,Y_\ell(\eta x)]$ $A = \frac{J_\ell(r_{\ell m})}{Y_\ell(r_{\ell m})}$ FOR TM MODES $r_{\ell m}$ = m^{th} ZERO OF $[J_\ell'(\eta x)\,Y_\ell'(x) - J_\ell'(x)\,Y_\ell'(\eta x)]$ $A = \frac{J_\ell'(r_{\ell m})}{Y_\ell'(r_{\ell m})}$ FOR TE MODES	$m > 0$ SPECIAL CASE OF TM 0,0,n MODE, WITH $r_{\ell m} = 0$
	TE			$m > 0$ $n > 0$

SOURCES: HANSEN, JNL. APP. PHYS., 9, P. 654 BORGNIS, HOCHF: TECH u. ELEK: AKUS., 56, P. 47

* THE TIME FACTOR HAS BEEN OMITTED;

CAVITY	MODE	NORMAL WAVELENGTHS	APPROXIMATION FOR TOTAL NUMBER OF MODES (TE & TM) HAVING $\lambda > \lambda_0$	FORMULAS FOR $Q\frac{\delta}{\lambda}$		DEFINITIONS
RECTANGULAR PRISM	TM	$\lambda = \frac{2}{\sqrt{\left(\frac{\ell}{a}\right)^2+\left(\frac{m}{b}\right)^2+\left(\frac{n}{L}\right)^2}}$	$N = 8.38\frac{V}{\lambda_0^3} - \frac{P}{\lambda_0}$ $V = abL$ $P = a+b+L$	$\frac{abL}{4}\cdot\frac{(p^2+q^2)(p^2+q^2+r^2)^{\frac{1}{2}}}{p^2b(a+L)+q^2a(b+L)}$	$n > 0$	$p = \frac{\ell}{a}$ $q = \frac{m}{b}$ $r = \frac{n}{L}$
				$\frac{abL}{2}\cdot\frac{(p^2+q^2)^{\frac{3}{2}}}{p^2b(a+2L)+q^2a(b+2L)}$	$n = 0$	
	TE	SAME AS TM MODES		$\frac{abL}{4}\cdot\frac{(p^2+q^2)(p^2+q^2+r^2)^{\frac{3}{2}}}{aL\left[p^2r^2+(p^2+q^2)^2\right]+bL\left[q^2r^2+(p^2+q^2)^2\right]+abr^2(p^2+q^2)}$	$\ell m > 0$	
				$\frac{abL}{2}\cdot\frac{(q^2+r^2)^{\frac{3}{2}}}{q^2L(b+2a)+r^2b(L+2a)}$	$\ell = 0$	
				$\frac{abL}{2}\cdot\frac{(p^2+r^2)^{\frac{3}{2}}}{p^2L(a+2b)+r^2a(L+2b)}$	$m = 0$	
CIRCULAR CYLINDER	TM	$\lambda = \frac{2}{\sqrt{\left(\frac{2r_{\ell m}}{\pi a}\right)^2+\left(\frac{n}{L}\right)^2}}$ $(fa)^2 = \left(\frac{cr_{\ell m}}{\pi}\right)^2 + \left(\frac{cn}{2}\right)^2\left(\frac{a}{L}\right)^2$ $c = \sqrt{\mu\epsilon}$ = VELOCITY OF ELECTROMAGNETIC WAVES IN DIELECTRIC f = FREQUENCY	$N = 4.38\frac{V}{\lambda_0^3} + 0.09\frac{S}{\lambda_0^2}$ $V = \frac{\pi a^2 L}{4}$ $S = \pi a L$	$\frac{r_{\ell m}}{2\pi}\left[1+p^2R^2\right]^{\frac{1}{2}}\cdot\frac{1}{1+R}$	$n > 0$	$R = \frac{a}{L}$ $p = \frac{n\pi}{2r_{\ell m}}$
				$\frac{r_{\ell m}}{\pi}\cdot\frac{1}{2+R}$	$n = 0$	
	TE			$\frac{r_{\ell m}}{2\pi}\left[1+p^2R^2\right]^{\frac{3}{2}}\frac{1-\left(\frac{\ell}{r_{\ell m}}\right)^2}{1+p^2R^3+p^2(1-R)R^2\left(\frac{\ell}{r_{\ell m}}\right)^2}$		
FULL COAXIAL	TM	SAME FORM AS FOR CYLINDER $r_{\ell m}$ HAS DIFFERENT VALUES	$N \approx 4.4\frac{V}{\lambda_0^3}$ WITH SOME DOUBT AS TO VALUE OF THE COEFFICIENT	$\frac{r_{\ell m}}{2\pi}\left[1+p^2R^2\right]^{\frac{1}{2}}\frac{(1-\eta^2H')}{(1+\eta H')+R(1-\eta^2H')}$	$n>0$	$R = \frac{a}{L}$ $p = \frac{n\pi}{2r_{\ell m}}$ $H' = \left[\frac{Z_\ell'(\eta r_{\ell m})}{Z_\ell'(r_{\ell m})}\right]^2$ $H = \left[\frac{Z_\ell(\eta r_{\ell m})}{Z_\ell(r_{\ell m})}\right]^2$
				$\frac{r_{\ell m}}{\pi}\cdot\frac{(1-\eta^2H')}{2(1+\eta H')+R(1-\eta^2H')}$ THESE EXPRESSIONS ARE NOT VALID FOR SMALL η WHEN $\ell = 0$	$n=0$	
	TE			$\frac{r_{\ell m}}{2\pi}\cdot\frac{\left[1+p^2R^2\right]^{\frac{3}{2}}M}{(1+\eta H)+p^2R^2\frac{\ell^2}{r_{\ell m}^2}\left(1+\frac{H}{\eta}\right)+p^2R^3M}$ WHERE $M = \left(1-\frac{\ell^2}{r_{\ell m}^2}\right)-\eta^2H\left(1-\frac{\ell^2}{\eta^2 r_{\ell m}^2}\right)$		

is confined to the circular cylinder, with changes in units and notation as specified later.

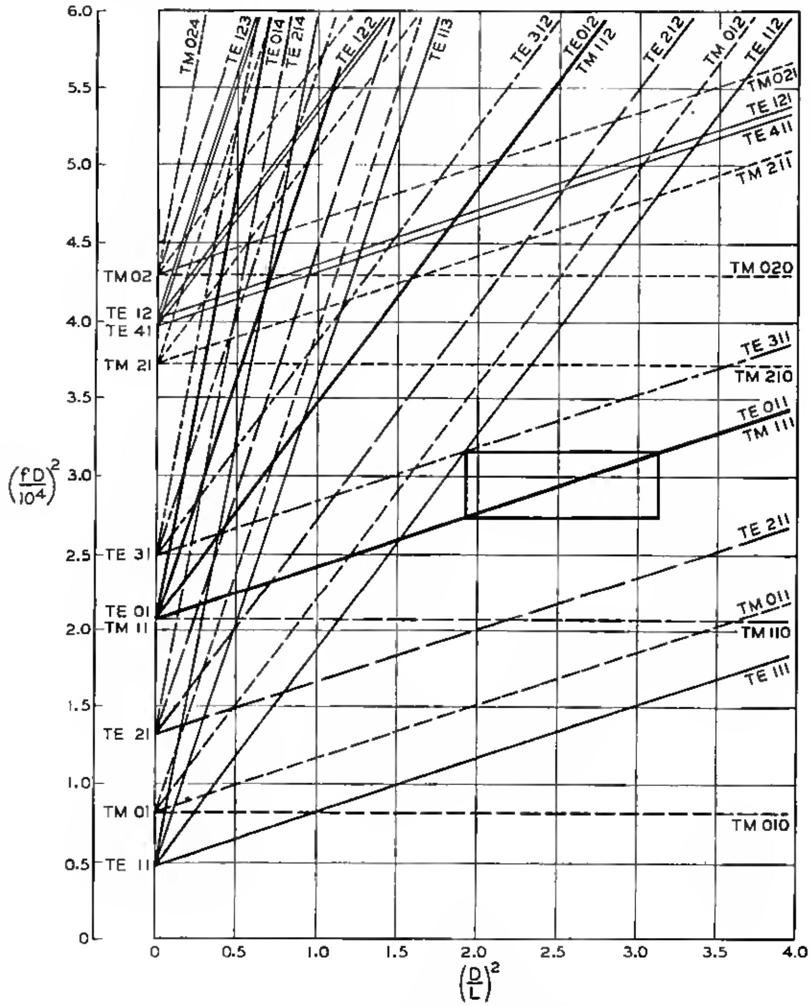


Fig. 1—Mode chart for right circular cylinder resonant cavity.

The Mode Chart

The formula relating the resonant frequency to the mode, shape and dimensions of the cylinder is of prime interest. It may be simply written as

$$(fD)^2 = A + Bn^2\left(\frac{D}{L}\right)^2 \tag{2}$$

where

f = frequency in megacycles per second.
D = diameter of cavity in inches.
L = length of cavity in inches.
A = a constant depending upon the mode. Values of A are given in Table II for the lowest 30 modes. Values of Bessel function roots are given in Table III for the first 180 modes.
B = a constant depending upon the velocity of electromagnetic waves in the dielectric. For air at 25°C and 60% relative humidity, $B = 0.34799 \times 10^8$.
n = third index defining the mode, i.e., the number of half wavelengths along the cylinder axis.

TABLE II.—*Constants for Use in Computing the Resonant Frequencies of Circular Cylinders*

$$(fD)^2 = \left(\frac{cr}{\pi}\right)^2 + \left(\frac{cn}{2}\right)^2\left(\frac{D}{L}\right)^2 = A + B\,n^2\left(\frac{D}{L}\right)^2$$

$B = 0.34799 \times 10^8$ $\qquad c = 1.17981 \times 10^{10}$ inches/second

Mode	r	A
TM 01	2.40483	0.81563×10^8
02	5.52008	4.2975
03	8.65373	10.5617
11	3.83171	2.0707
12	7.01559	6.9415
13	10.17347	14.5970
21	5.13562	3.7197
22	8.41724	9.9923
31	6.38016	5.7410
32	9.76102	13.4374
41	7.58834	8.1212
51	8.77148	10.8511
61	9.93611	13.9238
TE 01	3.83171	2.0707
02	7.01559	6.9415
03	10.17347	14.5970
11	1.84118	0.47810
12	5.33144	4.0088
13	8.53632	10.2770
21	3.05424	1.3156
22	6.70613	6.3426
23	9.96947	14.0175
31	4.20119	2.4893
32	8.01524	9.0606
41	5.31755	3.9879
42	9.28240	12.1520
51	6.41562	5.8050
61	7.50127	7.9359
71	8.57784	10.3772
81	9.64742	13.1265

Value of c is for air at 25°C. and 60% relative humidity. D and L in inches; f in megacycles.

Formula (2) represents a family of straight lines, when $(D/L)^2$ and $(fD)^2$ are used as coordinates, and leads directly to the easily constructed and highly useful "Mode Chart" of Fig. 1.

It will be noted from Tahle II that the *TE* $0mn$ and the *TM* $1mn$ modes have the same frequency of resonance. This is a highly important case of degeneracy. In the design of practical cavities it is necessary to take measures to eliminate this degeneracy, as the *TM* mode (usually referred to as the companion of its associated *TE* mode) introduces undesirahle effects. This is discussed more at length later.

Choice of Operating Mode

Turning now to the expressions for $Q\frac{\delta}{\lambda}$ these are seen to he of a rather complicated nature. For some of the lower order modes, their values are plotted in Figs. 2, 3 and 4. Examination of these leads to the question of which mode has the highest Q for a given volume. It is desirahle to keep the volume a minimum, since, as shown hy (1), the total number of resonances is a function of the volume. Analysis of the prohlem is somewhat involved, hut leads to the conclusion that operation in the *TE* $01n$ mode* gives the smallest volume for an assigned Q, and also leads to specific values of n and D/L which give this result. In fact, for maximum Q/V in the *TE* $01n$ mode,

$$(fD)^2\left(\frac{D}{L}\right) \doteq 3.11 \times 10^8 \tag{3}$$

which permits easy plotting on a mode chart of the locus of the operating points for hest Q/V ratio.

Extraneous or Unwanted Modes

In echo hoxes for radar testing, where high Q is of the utmost importance, the *TE* $01n$ mode has heen used. The values of n vary from 1 at frequencies around 1 *kmc* to 50 at ahout 25 *kmc*.

All other modes are then regarded as unwanted or extraneous. The great utility of the mode chart lies in that it permits a quick determination of the most favorahle operating area. We consider this now in detail.

Figure 5 shows a portion of a mode chart. It is clear that the sensihle way to construct a tunahle cylinder is to keep the diameter fixed and vary the length. With fixed diameter, the coordinates of the mode chart are essentially f^2 and $\left(\frac{1}{L}\right)^2$ and it is convenient to refer to them loosely as fre-

* Unimportant exceptions occur for values of $Q\frac{\delta}{\lambda} < 1.2$.

quency and length. With this understanding, if the frequency band to be covered by the tunable cavity extends from f_1 to f_2 then the length must be adjustable from L_1 to L_2. Responses to frequencies within the band, but

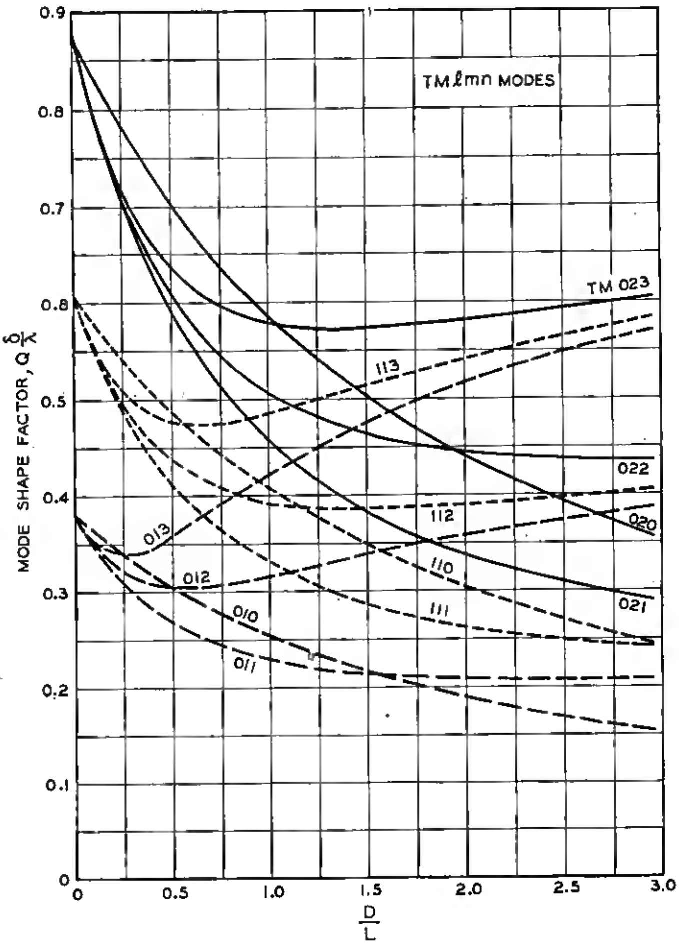


Fig. 2—Mode-shape factors $\left(Q\frac{\delta}{\lambda}\right)$ as a function of diameter to length ratio $\left(\frac{D}{L}\right)$ for circular cylinder resonator—TM modes.

which demand lengths outside the range L_1 to L_2 are of little interest because mechanical stops prohibit other lengths. On the assumption that the applied frequency will always lie within the operating band f_1 to f_2, responses to frequencies below f_1 or above f_2 are likewise of little interest. Therefore,

major consideration need be given only to those modes which lie within the rectangle of which the desired TE 01n mode forms the diagonal.

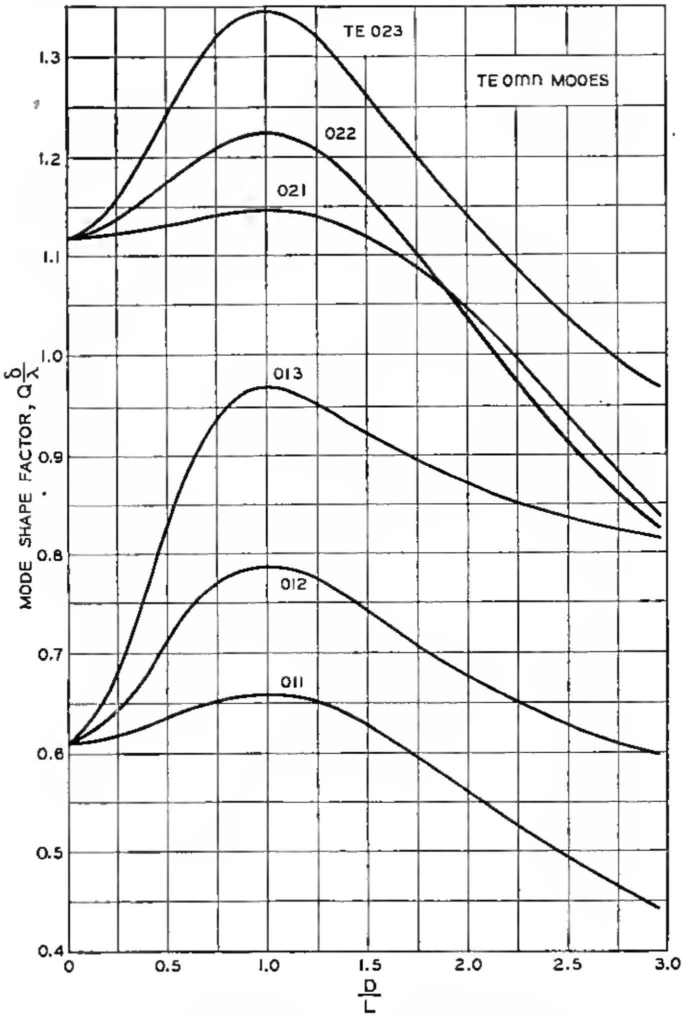


Fig. 3—Mode-shape factors $\left(Q\frac{\delta}{\lambda}\right)$ as a function of diameter to length ratio $\left(\frac{D}{L}\right)$ for circular cylinder resonator — TE modes with $l = 0$.

Any nondiagonal mode is an extraneous mode. Those which do not cross the desired mode within the frame are called interfering modes. They act to give responses at more than one tuning point when the applied fre-

quency is held fixed, or alternatively to give responses at more than one frequency when the tuning is held fixed. In either event they lead to ambiguity and confusion, and their effects must be reduced to the point where this cannot occur.

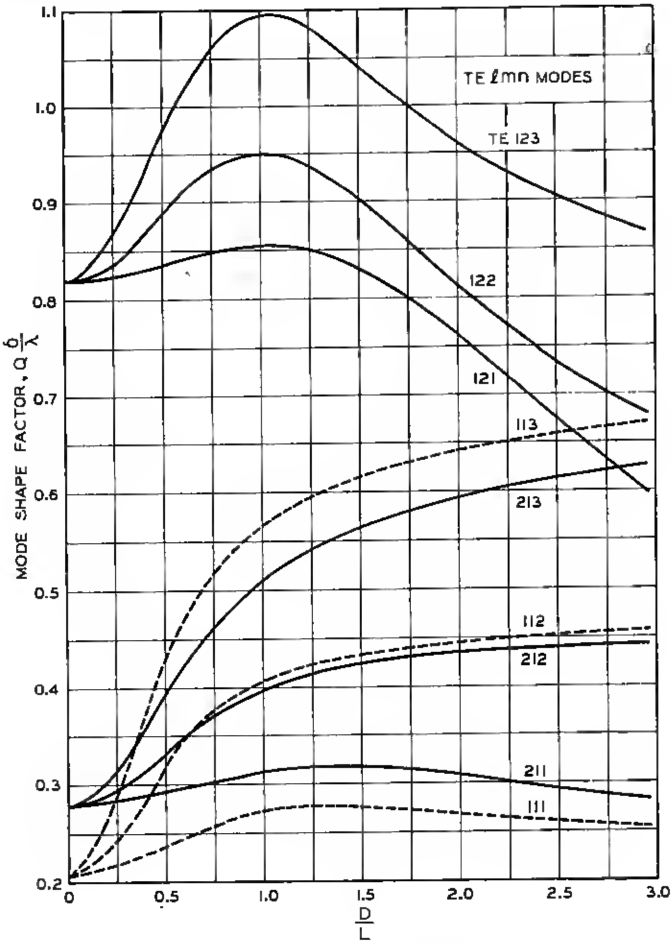


Fig. 4—Mode-shape factors $\left(Q\frac{\delta}{\lambda}\right)$ as a function of diameter to length ratio $\left(\frac{D}{L}\right)$ for circular cylinder resonator — TE modes with $l > 0$.

A special type of interfering mode is the $TE\ 01(n+1)$ mode. In the nature of things, it is virtually impossible to suppress this mode without likewise suppressing the desired $TE\ 01n$ mode. The width of the operating

band is tbus strictly limited, if ambiguity is to be avoided. This effect, termed self-interference, becomes an important factor as n increases, since

$$\frac{f_2}{f_1}\,(\text{maximum}) = \frac{n+1}{n}$$

This maximum value cannot be realized because it is incompatible with other requirements.

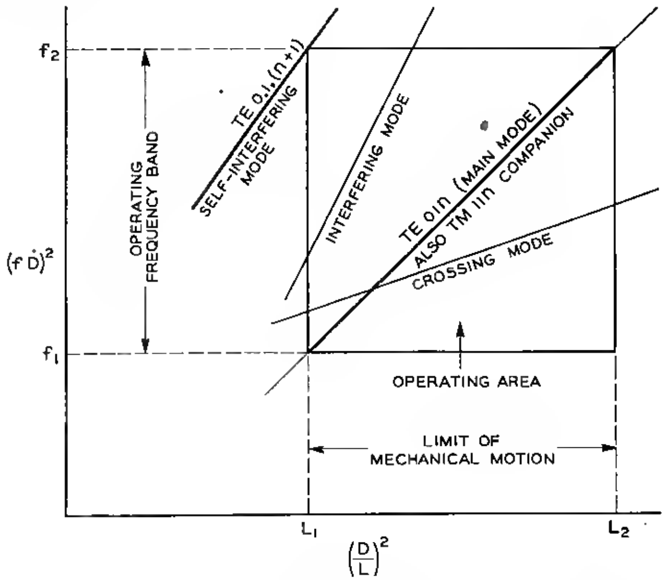


Fig. 5—Mode chart illustrating meaning of crossing and interfering modes and of operating area.

When an undesired mode crosses the main mode within the rectangle it is called a crossing mode. Except in a region close to the crossing point, it acts only to cause ambiguity as already discussed. In the immediate region of the crossing point, however, the cavity is simultaneously resonant in both modes. Violent interaction effects which may seriously degrade the cavity Q frequently occur at such a crossing.

Methods of Minimizing Effects of Unwanted Modes

A major problem in the design of a high Q cavity for radar test purposes, in which the Q and frequency range are set by radar system considerations, is to reduce the effects of all the undesired modes without seriously degrading the main *TE* 01n mode. Among those to be suppressed is the companion *TM* 11n mode which is inherently of the same frequency.

TABLE III.—*Values of the Bessel Function Zero (r_{lm}) for the First 180 Modes in a Circular Cylinder Resonator*

	r_{lm}	Mode*		r_{lm}	Mode*		r_{lm}	Mode*		r_{lm}	Mode*
1	1.8412	E 1-1	46	13.0152	M 3-3	91	18.4335	M 10-2	136	22.6716	E 2-
2	2.4048	M 0-1	47	13.1704	E 2-4	92	18.6374	E 6-4	137	22.7601	M 1-
3	3.0542	E 2-1	48	13.3237	M 1-4	93	18.7451	E 12-2	138	22.7601	E 0-
4	3.8317	M 1-1	49	13.3237	E 0-4	94	18.9000	M 14-1	139	22.9452	M 8-
5	3.8317	E 0-1	50	13.3543	M 9-1	95	18.9801	M 5-4	140	23.1158	M 14-
6	4.2012	E 3-1	51	13.5893	M 6-2	96	19.0046	E 9-3	141	23.2548	E 21-
7	5.1356	M 2-1	52	13.8788	E 12-1	97	19.1045	E 17-1	142	23.2568	M 18-
8	5.3176	E 4-1	53	13.9872	E 5-3	98	19.1960	E 4-5	143	23.2643	E 16-
9	5.3314	E 1-2	54	14.1155	E 8-2	99	19.4094	M 3-5	144	23.2681	E 7-
10	5.5201	M 0-2	55	14.3725	M 4-3	100	19.5129	E 2-6	145	23.2759	M 11-
11	6.3802	M 3-1	56	14.4755	M 10-1	101	19.5545	M 8-3	146	23.5861	M 6-
12	6.4156	E 5-1	57	14.5858	E 3-4	102	19.6159	M 1-6	147	23.7607	E 10-
13	6.7061	E 2-2	58	14.7960	M 2-4	103	19.6159	E 0-6	148	23.8036	E 5-
14	7.0156	M 1-2	59	14.8213	M 7-2	104	19.6160	M 11-2	149	23.8194	E 13-
15	7.0156	E 0-2	60	14.8636	E 1-5	105	19.8832	E 13-2	150	24.0190	M 4-
16	7.5013	E 6-1	61	14.9284	E 13-1	106	19.9419	E 7-4	151	24.1449	E 3-
17	7.5883	M 4-1	62	14.9309	M 0-5	107	19.9944	M 15-1	152	24.2339	M 9-
18	8.0152	E 3-2	63	15.2682	E 6-3	108	20.1441	E 18-1	153	24.2692	M 15-
19	8.4172	M 2-2	64	15.2867	E 9-2	109	20.2230	E 10-3	154	24.2701	M 2-
20	8.5363	E 1-3	65	15.5898	M 11-1	110	20.3208	M 6-4	155	24.2894	E 22-
21	8.5778	E 7-1	66	15.7002	M 5-3	111	20.5755	E 5-5	156	24.3113	E 1-
22	8.6537	M 0-3	67	15.9641	E 4-4	112	20.7899	M 12-2	157	24.3382	M 19-
23	8.7715	M 5-1	68	15.9754	E 14-1	113	20.8070	M 9-3	158	24.3525	M 0-
24	9.2824	E 4-2	69	16.0378	M 8-2	114	20.8269	M 4-5	159	24.3819	E 17-
25	9.6474	E 8-1	70	16.2235	M 3-4	115	20.9725	E 3-6	160	24.4949	M 12-
26	9.7610	M 3-2	71	16.3475	E 2-5	116	21.0154	E 14-2	161	24.5872	E 8-
27	9.9361	M 6-1	72	16.4479	E 10-2	117	21.0851	M 16-1	162	24.9349	M 7-
28	9.9695	E 2-3	73	16.4706	M 1-5	118	21.1170	M 2-6	163	25.0020	E 14-
29	10.1735	M 1-3	74	16.4706	E 0-5	119	21.1644	E 1-7	164	25.0085	E 11-
30	10.1735	E 0-3	75	16.5294	E 7-3	120	21.1823	E 19-1	165	25.1839	E 6-
31	10.5199	E 5-2	76	16.6982	M 12-1	121	21.2116	M 0-7	166	25.3229	E 23-
32	10.7114	E 9-1	77	17.0038	M 6-3	122	21.2291	E 8-4	167	25.4170	M 16-
33	11.0647	M 4-2	78	17.0203	E 15-1	123	21.4309	E 11-3	168	25.4171	M 20-
34	11.0864	M 7-1	79	17.2412	M 9-2	124	21.6415	M 7-4	169	25.4303	M 5-
35	11.3459	E 3-3	80	17.3128	E 5-4	125	21.9317	E 6-5	170	25.4956	E 18-
36	11.6198	M 2-3	81	17.6003	E 11-2	126	21.9562	M 13-2	171	25.5094	M 10-
37	11.7060	E 1-4	82	17.6160	M 4-4	127	22.0470	M 10-3	172	25.5898	E 4-
38	11.7349	E 6-2	83	17.7740	E 8-3	128	22.1422	E 15-2	173	25.7051	M 13-
39	11.7709	E 10-1	84	17.7887	E 3-5	129	22.1725	M 17-1	174	25.7482	M 3-
40	11.7915	M 0-4	85	17.8014	M 13-1	130	22.2178	M 5-5	175	25.8260	E 2-
41	12.2251	M 8-1	86	17.9598	M 2-5	131	22.2191	E 20-1	176	25.8912	E 9-
42	12.3386	M 5-2	87	18.0155	E 1-6	132	22.4010	E 4-6	177	25.9037	M 1-
43	12.6819	E 4-3	88	18.0633	E 16-1	133	22.5014	E 9-4	178	25.9037	E 0-
44	12.8265	E 11-1	89	18.0711	M 0-6	134	22.5827	M 3-6	179	26.1778	E 15-
45	12.9324	E 7-2	90	18.2876	M 7-3	135	22.6293	E 12-3	180	26.2460	E 12-

* Nomenclature after Barrow & Mieher, "Natural Oscillations of Electrical Cavity Resonator IRE Proceedings, April 1940, p. 184. M modes take zeros of $J_l(x)$; E modes take zeros of $J_l'($. Number directly following E or M is l; number after hyphen is number of root.

Values less than 16.0 are abridged from six-place values and are believed to be correct; valu more than 16.0 are abridged from five-place values and may be in error by one unit in fourth decin place. **5** in fourth place indicates that higher value is to be used in rounding off to fewer decima

One solution, of limited application, is to choose an operating rectangle free from extraneous responses. An alternative solution is to design the cavity in a manner such that the undesired responses are reduced or sup-

pressed to a point wbere their presence does not interfere with the normal operation of tbe cavity. In tbis latter case, the amount of suppression is naturally dependent upon the use to which the cavity is to be put, and is conceivably different for a high Q cavity used as a frequency meter, for example, and one used as a selective filter.

Experience has shown that certain families of modes are much more difficult to suppress than others, and are to be avoided, if at all possible. The feasibility of doing this can be determined by sliding the operating area (a suitable opening in a sheet of paper) around on a large mode chart until the most favorable operating region, consistent witb otber requirements, has been found.

Once a suitable operating area has been chosen, the cavity diameter is fixed and length and frequency scales added to the mode chart make it read directly in quantities readily measured.

Cavity Couplings

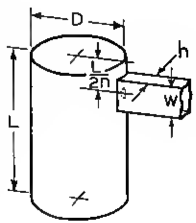
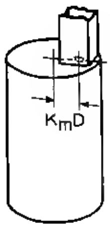
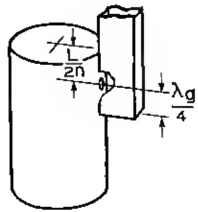
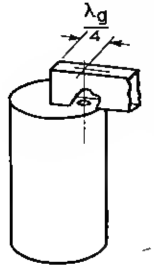
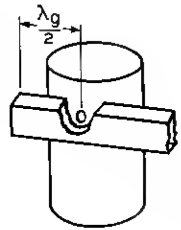
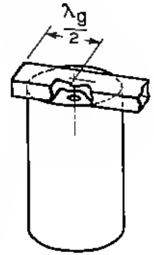
To be useful tbe cavity must be coupled to external circuits. Tbe problem here is to get the correct coupling to the main mode and as little coupling as possible to all others. Since the electric field is zero everywhere at the boundary surface of the cavity for the *TE* 01*n* mode, coupling to it must be magnetic. This may be obtained either by a loop at the end of a coaxial line or hy an orifice connecting the cavity with a wave guide.

The location for maximum coupling to the main mode is on the side of the cavity, an odd number of quarter-guide wavelengths from the end, or on the end about halfway (48%) out from the center to the edge. Correct orientation of loop or wave guide is achieved when the magnetic fields are parallel. This requires the axis of the loop to be parallel to the axis of tbe cylinder for side wall feed and to be perpendicular to the cylinder axis for end feed. Wave guide orientation is shown in Table IV.

The theory of coupling loops and orifices is not at present precise enougb to yield more than approximate dimensions. Exact sizes of loops and boles have therefore been obtained experimentally for all designs.

On the hasis of ratber severely limiting assumptions,[11] coupling formulas for a round hole connecting a rectangular wave guide and a *TE* 01*n* cavity are given in Table IV. The assumptions are that tbe orifice is in a wall of negligible thickness, its diameter is small compared to the wavelengtb, it is not near any surface discontinuity, and tbat the wave guide propagates only its principal (gravest) mode and is perfectly terminated. In echo box applications, this theory leads to a computed diameter that is somewhat smaller than experiment shows to be correct.

The coupling to other modes can be analyzed, at least qualitatively, from the field expressions of Tahle I. This has heen of value in making final

	CASE					
	1A	1B	2A	2B	3A	3B
COUPLING METHOD						
CIRCULAR ORIFICE	$\frac{\Delta f}{f} = -K_c \frac{\lambda^2 d^3}{D^4 L}$ $W_a = K_w \frac{\lambda^2 d^6}{\lambda_g w h D^4 L}$	$\frac{\Delta f}{f} = -K_c \frac{n^2 \lambda^2 d^3}{D^2 L^3}$ $W_a = K_w \frac{n^2 \lambda^2 d^6}{\lambda_g w h D^2 L^3}$	$\frac{\Delta f}{f}$ SAME AS 1A $W_a = K_w \frac{\lambda_g \lambda^2 d^6}{w^3 h D^4 L}$	$\frac{\Delta f}{f}$ SAME AS 1B $W_a = K_w \frac{n^2 \lambda_g \lambda^2 d^6}{w^3 h D^2 L^3}$	$\frac{\Delta f}{f}$ SAME AS 1A W_a SAME AS 1A	$\frac{\Delta f}{f}$ SAME AS 1B W_a SAME AS 1B
CONSTANTS	K_c K_w TE 01n 0.316 1.322 TE 02n 1.058 4.43 TE 03n 2.225 9.32	K_c K_w K_m 0.1107 0.464 0.2403 0.1995 0.836 0.1312 0.288 1.207 0.0905	K_w 0.331 1.108 2.330	K_w 0.1159 0.2089 0.302		

NOTATION: λ = FREE SPACE WAVELENGTH OF CAVITY RESONANCE d = DIAMETER OF ORIFICE

λ_g = GUIDE WAVELENGTH $W_a = \frac{1}{Q_a}$ = CAVITY LOADING

NOTE: FOR FEED LIKE CASES 2 AND 3, BUT WITH WAVEGUIDE TERMINATED IN BOTH DIRECTIONS, DIVIDE W_a BY 2

TABLE IV.—*Orifice Coupling of Wave Guide (TE 10 Mode) to Cylindrical Cavity (TE 0mn Mode)*

small relocations of the coupling points to discriminate against a residual undesired mode.

Principle of Similitude

One other theorem generally applicable to all cavities has been useful in design. It is the principle of similitude, which may be stated as follows[12]:

> A reduction in all the linear dimensions of a cavity resonator by a factor $1/m$ (if accompanied by an increase in the conductivity of the walls by a factor m) will reduce the wavelengths of the modes by a factor $1/m$.

The condition given in parentheses is necessary for strict validity; for high Q cavities, it need not be considered.

Application of Theory

An illustration of an engineering application of the basic information just presented, is the design of an echo box test set for use in radar maintenance.

The test set has a number of components, but only the cavity proper and its couplings will be considered at this time.

Design Requirements

The basic design requirements of the cavity set by the radar are: (1) the working decrement, and (2) the tunable frequency limits (f_1 and f_2). Service use of these test sets in the 3 *kmc* and 9 *kmc* bands has shown that a working decrement of about 3 *db* per microsecond gives satisfactory results.

As seen in the discussion above, Q is a more useful design parameter for the resonant cavity than decrement, d. Hence the conversion

$$Q = \frac{27.3f}{d} \tag{4}$$

where d is expressed in *db* per microsecond, and f is in megacycles, gives the loaded or working Q to be realized.

Determining the Theoretical Q Required

For design purposes, however, there are several factors which dictate the use of a value for theoretical Q which is somewhat higher than the loaded Q just computed. The input and output couplings reduce the theoretical Q of the cavity due to their loadings. The coupling factor, s, expressed as the ratio of loaded to unloaded Q, has values for echo boxes of about 0.90 for the input coupling and from 0.90 to 0.99 for the output coupling. In addition, other factors such as the means used for mode suppression may degrade

tbe Q. But, in simple designs, these may be negligible. Therefore, it is expedient to design for a theoretical Q of about 15 to 25 per cent in excess of the working Q to be realized.

The working Q's of a number of echo box designs are cited here to indicate the order of magnitude required at several frequencies:

1 *kmc*	70,000*
3	40,000
9	100,000
25	200,000

Finding the Cavity Dimensions

With the frequency and theoretical Q known, the dimensions of the cavity can be evaluated but the formulas of Table I require some simplification for engineering use.

The mode-shape (MS) factor, $Q\frac{\delta}{\lambda}$, may also be termed its selectivity which for the TE $01n$ modes may be expressed as follows:

$$Q\,\frac{\delta}{\lambda} = 0.610\,\frac{\left[1 + 0.168\left(\frac{D}{L}\right)^2 n^2\right]^{3/2}}{1 + 0.168\left(\frac{D}{L}\right)^3 n^2} \tag{5}$$

where:

$\delta =$ the skin deptb in cm. $= \frac{1}{2\pi}\sqrt{\frac{10^9\rho}{f}}$

$\rho =$ the resistivity in obm-cm.

The skin depth is a factor wbicb recognizes the dissipation of energy in the walls and ends of the cylinder. With increase of resistivity of tbese surfaces the currents penetrate deeper and the resulting Q is lower.

A comparison of the relative Q's computed from tbe resistivity of several metals will show the importance of tbis factor:

Silver	1.03
Copper	1.00
Gold	0.84
Aluminum	0.78
Brass	0.48

Therefore, a brass cavity will have about one-half of the Q that a similar cavity would have if made of copper. Similarly, the silverplating of a copper cavity will gain about 3 per cent in Q.

Equation 5 may be made more convenient for calculations by combining

* This value reflects the higher Q required on ground radars.

terms which are a function of frequency and by assuming the conductivity of copper* for the cylinder walls. It then becomes

$$\frac{Q\sqrt{f}}{10^6} = 2.77 \frac{\left[1 + 0.168\left(\frac{D}{L}\right)^2 n^2\right]^{3/2}}{1 + 0.168\left(\frac{D}{L}\right)^3 n^2} \tag{6}$$

for $TE\ 01n$ modes; f is in megacycles.

Thus, it is seen that for the design parameters Q and f, selected values of n now define tentative useful points on the mode chart in terms of D/L and n.

Selection of Operating Area on Mode Chart

This will be more evident upon examination of Fig. 6, which is a basic design chart for cylindrical cavity resonators using $TE\ 01n$ modes. The coordinates of $(fD)^2$ and $(D/L)^2$ will be recognized from the previous discussion of the mode chart (Fig. 1) although in this case the range has been expanded by the use of logarithmic coordinates. Mode identification is obtained from equation 2; which, for $TE\ 01n$ modes becomes

$$(fD)^2 \times 10^{-8} = 2.0707 + 0.3480\, n^2\, (D/L)^2 \tag{7}$$

with D and L in inches and f in megacycles.

A family of $TE\ 01n$ modes has been drawn on the chart for selected values of n. To aid in designing for minimum volume a line labelled Max. $\frac{Q}{V}$ has been added (Equation 3). Lines of constant $Q\sqrt{f}$ are also shown as a series of dashed lines.

A tentative operating area on this chart may be selected on the basis of the required $Q\sqrt{f}$. Using mid-frequency for f, the intersection of the $Q\sqrt{f}$ line and the minimum volume line will define the operating mode, n, and also locate the center of the operating rectangle.

An enlarged plot of this area as in Fig. 7 will show all modes possible in the cavity. Some adjustment of the precise location of this area may then be desirable to eliminate certain types or numbers of unwanted modes. For example, it is extremely difficult to suppress $TE\ 02$ modes without affecting the $TE\ 01$ mode, because of the very close resemblance of the field configurations. On the other hand, most TM modes are easy to handle.

It may be possible to select an operating area such as the rectangle blocked out in Fig. 1 in which all extraneous responses (with the exception of the companion $TM\ 111$) are avoided. The largest rectangle which can be inscribed here is limited by the $TE\ 311$ and $TE\ 112$ modes. This will

* $\rho = 1.7241 \times 10^{-6}$ ohm-cm—the International Standard value for copper.

permit a ±3.6 per cent frequency range. Several designs of 3 *kmc* echo boxes have been based on this area.

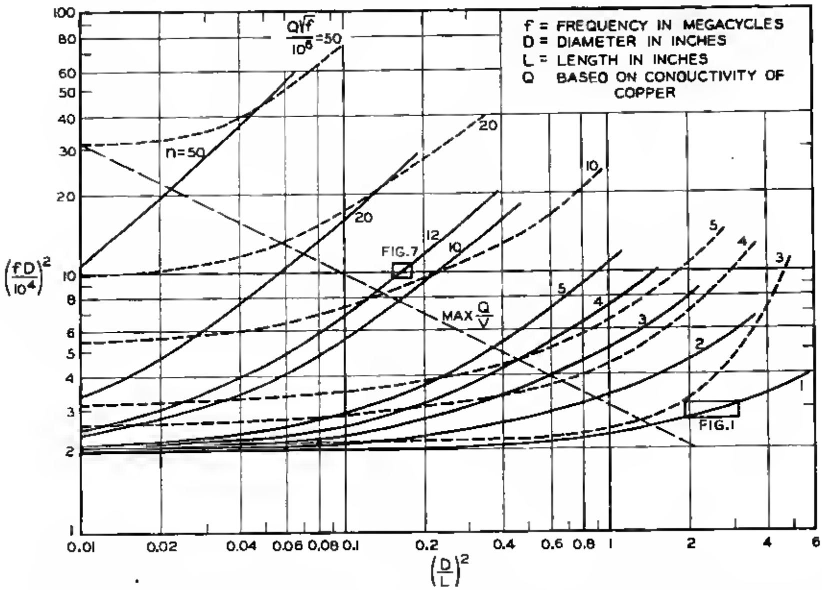


Fig. 6—Design chart for cylindrical cavity resonators operating in the *TE* 01*n* modes.

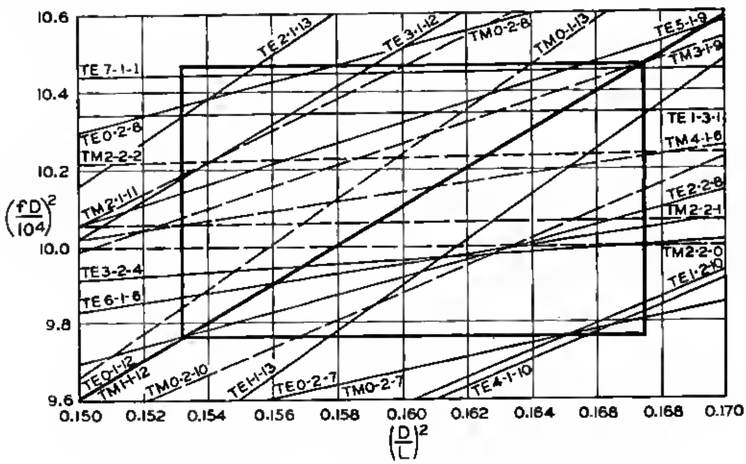


Fig. 7—Expanded mode chart covering the operating area of a 9 *kmc* echo box.

This type of solution is quite adequate for many of the simpler designs; but as the required Q becomes larger or the frequency coverage becomes

greater it is generally not possible to locate such areas. An operating area for 9 *kmc* is shown on Fig. 7. Nine crossing modes and twelve interfering modes exist. For 25 *kmc* the crossing modes run into tbe forties with hundreds of interfering modes.

Suppression of the undesired modes requires a thorough knowledge of their field configurations and a number of effective techniques which may be applied on a practical engineering basis. Several examples are cited in later sections.

Since decrement is an important characteristic of these cavities, especially when applied to radar test sets, the uniformity of the decrement over the frequency range or "flatness of response" may be a significant design requirement. It will be seen from Fig. 3, that the MS factor of the wanted mode is not constant with varying D/L. In fact, if it were, tbe decrement would increase as the 3/2 power of frequency.

There are at least three attacks on this "flatness" problem: (1) to operate on the sloped portion of the MS curve in such a manner that its characteristic will tend to be complementary to the change with frequency; (2) to obtain compensation by varying the coupling with frequency—generally accomplished by selecting an appropriate coupling point along the side wall of the cavity; and (3) to overplate a portion of the cylinder's interior surface with a material of higher resistivity such as cadmium. For this third method, the formulas for $Q\frac{\delta}{\lambda}$ of Table I are no longer applicable since they assume a uniform resistivity of the cavity walls.

Thus, it will be seen that the final design of a cavity resonator is a compromise between a number of desired characteristics:

a) A cavity of minimum volume for a given Q.

b) A cavity having a minimum of extraneous responses of types difficult to suppress.

c) A cavity with compensation for flatness of decrement.

Engineering judgment is required to weigh the emphasis on each of these requirements which at times may be mutually exclusive.

Some Practical Considerations

Physically realizing the theoretical characteristics just described to obtain a satisfactory cavity brings forth a host of practical design problems. A number of these will be discussed in this section.

Description of Echo Box Test Sets

The schematics (Figs. 8 and 9) and photographs (Figs. 12 to 14) show the components and various construction methods of echo boxes in the 3 and 9 *kmc* bands. The cavity itself may be spun, drawn or turned of material

such as brass or aluminum which will give mechanical stability without excessive weight.

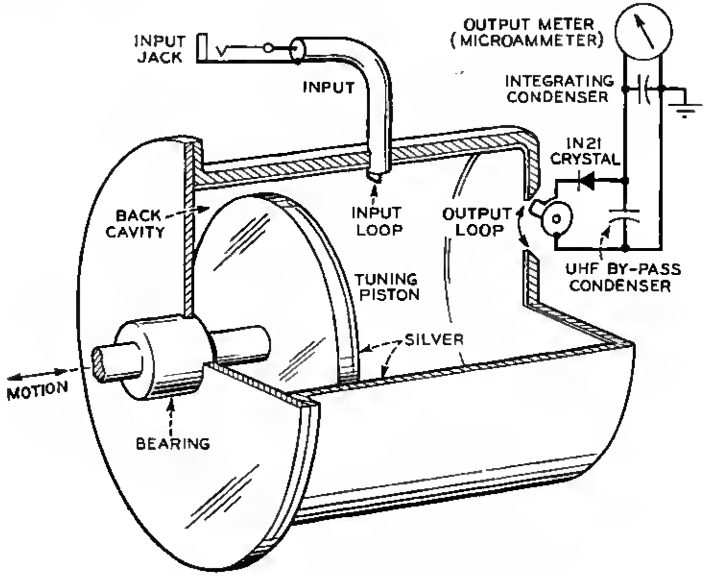


Fig. 8—Schematic of a 3 *kmc* echo box.

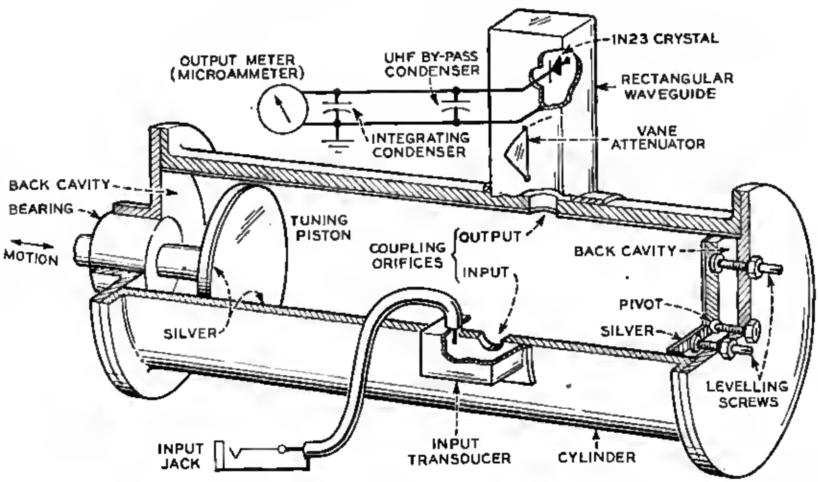


Fig. 9—Schematic of a 9 *kmc* echo box.

The movable plate is driven by the piston which operates in a close-fitting bearing. The drive mechanism translates the rotary motion of the tuning

knob into linear motion of the piston through a crank and connecting rod assembly. Coupled to the drive shaft is an indicating dial to register frequency.

Silverplating is indicated on all interior surfaces of the cavity for minimum resistance to currents in the walls and ends of the cylinder.

For the 3 *kmc* bands, the input coupling is in the form of a loop protruding into the cavity and connected by coaxial microwave cable to the radar under

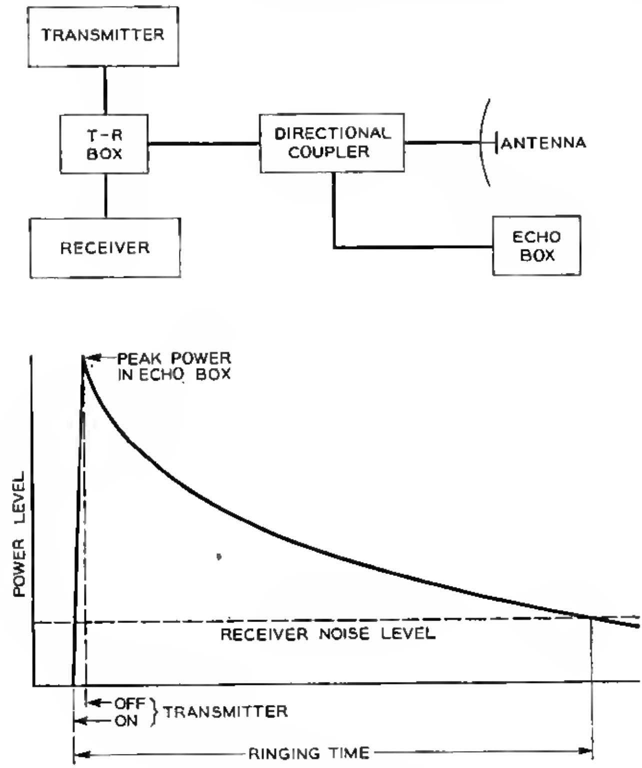


Fig. 10—Radar test with echo box.

test. For the 9 *kmc* bands, the input coupling is an orifice with which is associated a wave guide to coaxial cable transducer.

The output couplings for the two frequency bands also differ in construction. For the 3 *kmc* bands, the variable coupling is achieved by rotating the output loop before an aperture in the cavity. In this rotary mount is housed the rectifying crystal and by-pass condenser. An orifice is used for the 9 *kmc* band output coupling which feeds a crystal mounted in a wave guide. Amplitude control is by a vane attenuator in the wave guide. In

all cases an integrating condenser is required to smooth out the *d-c* pulses delivered by the crystal to give output indication on the *d-c* microammeter.

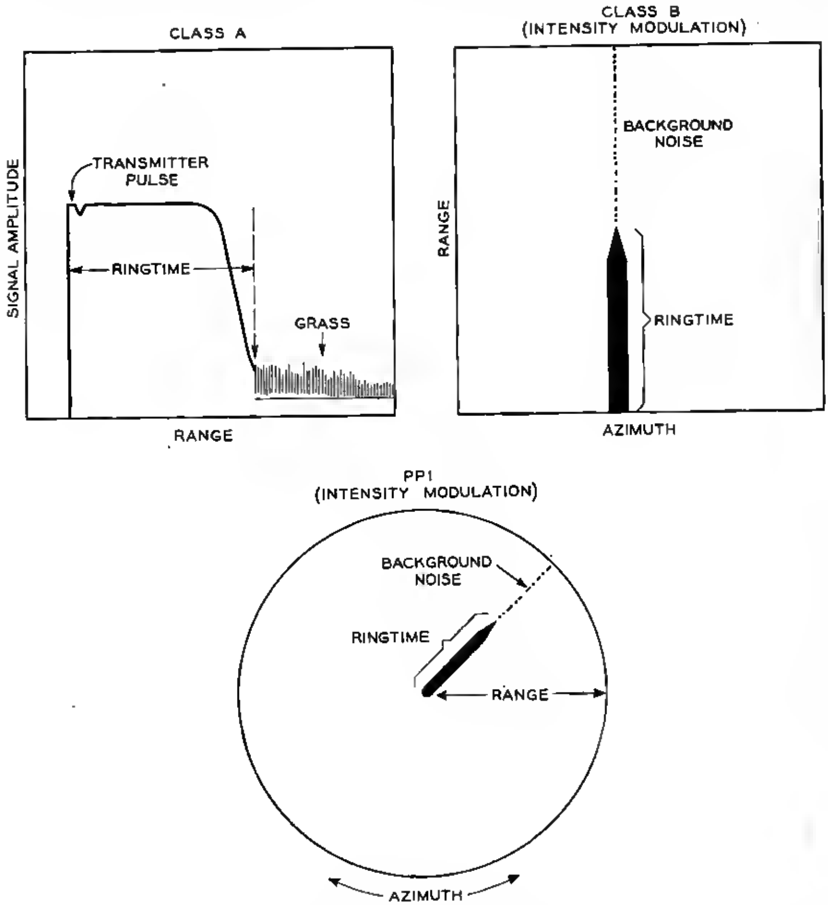


Fig. 11.—Typical ringtime patterns on radar indicators.

The cylinder in the 9 *kmc* band is an assembly of a tube and two end plates. One of these is driven by the piston as described and the other is adjustable for "levelling" i.e., parallelism of the two plates.

End Plate Gaps

The wanted *TE* mode is paired with a companion unwanted *TM* as described above. The fact that the *TM* mode has a Q substantially less than that of the *TE* makes the realization of the higher Q difficult. A method is

needed of suppressing the *TM* in the presence of the *TE*. Since the resonant cavity is a cylinder, of variable length, the movable end plate (or reflector)

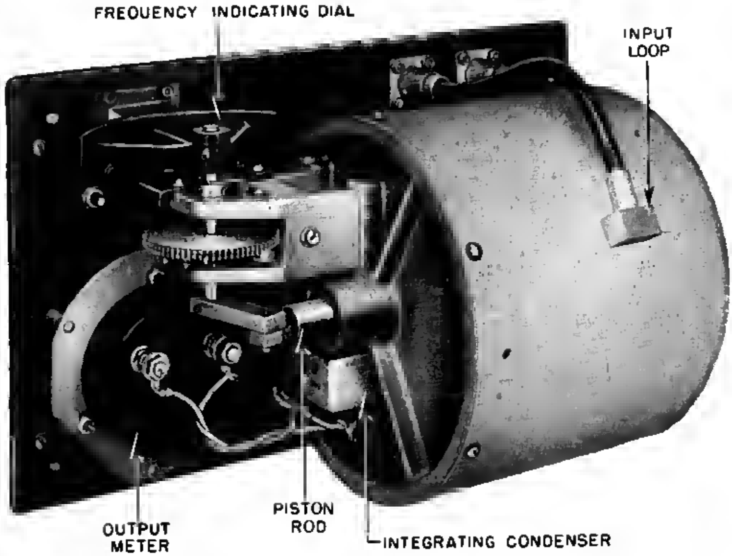


Fig. 12—Type of construction in a 3 *kmc* echo box.

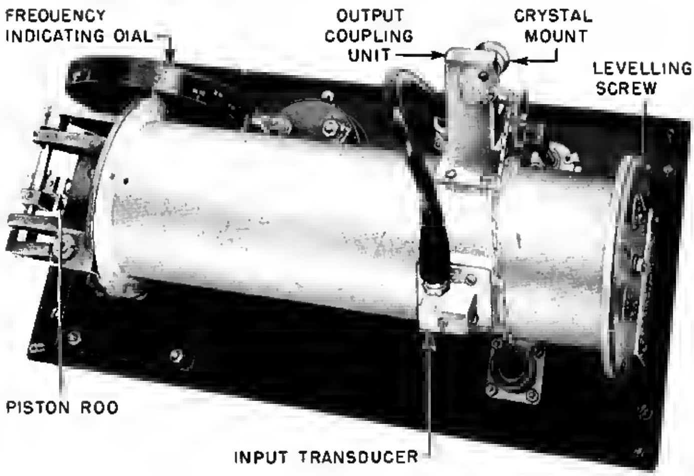


Fig. 13—Type of construction in a 9 *kmc* echo box.

can be modified to include a gap at its periphery. The gap perturbs the resonant frequencies of the two modes by different amounts so that they

become separated. Secondly, the peripheral gap cuts through the surface currents at points of high density for *TM* modes and minimum density of *TE* $0mn$ modes and hence is a form of mode suppression. Thirdly, the gap greatly simplifies the mechanical design of a movable end plate by eliminating the need of physical contact with the side wall of the cylinder.

A similar gap may be used at the other end. This facilitates "levelling" of a false bottom in the cavity.

Fig. 14—Panel view of a 9 *kmc* echo box.

Back Cavity Effects

The cavity with a peripheral gap may give rise to further spurious resonances in the region behind the reflecting surface (known as the back cavity) if these responses are not damped. The addition of a lossy material such as bakelite or carbon loaded neoprene in the back cavity is a successful suppression method.

Cylinder Tolerances

The geometry of the structure is very important in realizing the potential Q of the cavity. The theoretical computations are based on a perfect right circular cylinder which in practice is seldom achieved. Distortions occur in various forms: e.g., the cylinder instead of being round may be elliptical; the ends may not be perpendicular to the axis of the cylinder or not parallel to each other; surface irregularities may be present causing field distortions within the cavity. Many of these effects have been minimized by requiring adherence to close dimensional tolerances.

For some designs, the requirement on the parallelism of the end plates is greater than can be commercially produced. An adjustable mechanism for

levelling is a practical solution. Tilt adjustments in the order of 0.001 inch at the edge of the plate (about 3-inh diameter) are required in the 9 *kmc* band. Experience in the 25 *kmc* band shows evidence of the need for even finer control.

Plating

In addition, adequate control of the conductivity of the interior surfaces of the cavity is necessary to achieve a uniform manufactured product. This requires attention not only to the thickness and uniformity of the plating but also to the purity of the plating baths and the avoidance of introduction of foreign matter during buffing processes.

Couplings

The type and location of the coupling means can be used to discriminate between wanted and unwanted modes. Hence, this is a fertile field for mode suppression techniques. For example, since *TM* modes have $H_z = 0$, orifice coupling to the main mode at the side wall of the types shown in Table IV, cases 1A and 2A, will not couple to any *TM* modes. Again, if end coupling is used in a cavity which will support hoth the *TE* 01 and *TE* 02 modes, hy locating this coupling at the point where $H_\rho = 0$ for the *TE* 02 mode (about 54% of the way out from the center), it will not be excited and coupling to the *TE* 01 will be only slightly below maximum.

For echo box test sets the magnitude of the input coupling to the wanted mode is a compromise between the incomplete buildup of the fields within the cavity during the charging interval and the loading of the cavity Q on discharge. This is carried on by varying the coupling and observing the "ringtime" (the echo box indication on the radar scope). Optimum coupling is achieved when ringtime is made a maximum.

Output couplings for echo boxes are made so that just enough energy is withdrawn from the cavity to give an adequate meter reading.

Drive Mechanism

The objective of the design of the tuning mechanism is to provide a smooth, fine control with a minimum of hacklash. An illustration of the mechanical perfection required can he cited in a 9 *kmc* band design where $\frac{1}{4}$ inch of travel covered 200 megacycles in frequency. Hence, for frequency settings to he reproducihle to within $\frac{1}{4}$ mc the mechanical backlash of the moving parts had to be held to about 0.0003 inches or 0.3 mil. To realize this in commercial manufacture and to maintain it after adverse operating conditions such as vibration and shock was a major mechanical design problem.

In the design of this drive mechanism it should he recognized tbat equal

increments of cavity length will not produce uniform increments in frequency. The mode chart indicates graphically that a straight-line relationship exists between $(f)^2$ and $\left(\frac{1}{L}\right)^2$. Uniformly spaced markings on a dial reading directly in frequency can be realized by the use of such mechanisms as an eccentric operating on a limited arc. Adjustments are customarily provided to bring the cavity resonance and dial indication into agreement at some frequency of test. Frequency departures of the drive mechanism referred to this point are held commercially to about one part in 5000.

Application of Similitude

Echo box developments have often been undertaken at frequencies where adequate test equipment was not available. This has been especially true as the radar art progressed to higher and higher frequencies.

The principle of similitude has been utilized in the construction and test of models at the frequency of existing test facilities. The models have then been scaled to the assigned frequency band. This has been found to be a very practical expedient.

Use of Cavities For Radar Testing

The high Q resonant cavity when appropriately connected to a radar system returns to it a signal which may be used to judge the over-all performance of the radar. Its operation is as follows: During the transmitted pulse, microwave energy from the radar is stored in the cavity in the electromagnetic field. The charge of the cavity increases exponentially during this interval but fails to reach saturation for the cavity by a substantial margin because the pulse is too short. At the end of the pulse, the decay of this field supplies a signal of the same frequency as that of the radar transmitter (when the echo box is in tune) which is returned to the radar receiver as a continuous signal diminishing exponentially in amplitude.

The time interval between the end of the transmitted pulse and the point where the signal on the radar disappears into the background noise is the "ringtime." The term is used somewhat loosely since, in actual practice, the ringtime is measured on indicators whose range markings are generally in miles or yards referred to the beginning rather than the end of the pulse. The difference, of course, is small. It is customary to include the pulse length in all ringtime figures on operating radar systems. Typical ringtime patterns on radar indicators and a schematic of a radar test with an echo box are shown in Figs. 10 and 11.

As the output power of the radar either increases or decreases corresponding changes in the "charge" of the cavity will be reflected directly in ringtime changes. Similarly as the noise level of the radar receiver varies the

merging point of the cavity signal and the noise will show proportional changes in ringtime. Hence, the ringtime indication measures these two factors on which the radar's ability to discern real targets so largely depends.

The exponential buildup and decay of the charge in the cavity occur at a rate determined by the working decrement of the cavity. As mentioned previously, a decrement of about 3 *db* per microsecond is a satisfactory value for the 3 *kmc* and 9 *kmc* bands. A one microsecond change in ringtime (roughly one-tenth mile) would, therefore, represent a change in system performance of 3 *db*.

Uniformity Control and Expected Ringtime

By introducing an adjustment for the working Q of the cavities it is now possible to control the uniformity of the manufactured product to very close limits. Other improvements have also been incorporated which insure that boxes which have been made alike as to Q will similarly give uniform ringtime indications on a test radar. If the test sets are all alike as to ringtime, it is then possible to quote an "expected ringtime" for each of the various radars to be serviced by the echo box. Initially a measuring tool indicating relative changes in day to day operation of the radars, the uniformity provision with its "expected ringtime" has made the echo box test set an absolute measuring instrument of moderate precision.

Other Uses

In addition to its use as a measure of over-all performance of a radar, a significant number of diagnostic tests may be performed when trouble develops, which aid in rapidly locating the source. One such test is spectrum analysis. The extreme selectivity of the high Q cavity permits examination of the spectrum of the pulsed wave and from this may be deduced characteristics of the pulse, including pulse length. Multiple-moding of the magnetron circuit is easily shown by this analysis.

The meter of the test set gives a relative indication of the output power of the radar and this in itself assists greatly in segregating transmitter troubles from receiver troubles.

Also of importance is the use of the echo box as a frequency meter. The high Q of the cavity plus the fine control of the drive mechanism and the direct reading dial give excellent results (comparable to that of a wavemeter).

Acknowledgement

The design of resonant cavities is a difficult and complex art. In bringing it to the present state, the number of individuals who have made significant mathematical, theoretical, engineering and mechanical contributions is so

large that it is impossible to give them due credit by name. To them, however, the authors wish to acknowledge their indebtedness and to express their appreciation.

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